1

$$\begin{aligned}{}^5C_0 &= \frac{5!}{0!\cdot(5-0)!}\\ &= \frac{5!}{0!\cdot 5!}\\ &= 1\end{aligned} \qquad \begin{aligned}{}^5C_1 &= \frac{5!}{1!\cdot(5-1)!}\\ &= \frac{5!}{1!\cdot 4!}\\ &= \frac{5\cdot 4!}{1!\cdot 4!}\\ &= \frac{5\cdot 4}{1}\\ &= 5\end{aligned}$$

$$\begin{aligned}{}^5C_2 &= \frac{5!}{2!\cdot(5-2)!}\\ &= \frac{5!}{2!\cdot 3!}\\ &= \frac{5\cdot 4\cdot 3!}{2!\cdot 3!}\\ &= \frac{5\cdot 4}{2}\\ &= 10\end{aligned} \qquad \begin{aligned}{}^5C_3 &= \frac{5!}{3!\cdot(5-3)!}\\ &= \frac{5!}{3!2!}\\ &= \frac{5\cdot 4\cdot 3!}{3!\cdot 2!}\\ &= \frac{5\cdot 4}{2}\\ &= 10\end{aligned}$$

$$\begin{aligned}{}^5C_4 &= \frac{5!}{4!\cdot(5-4)!}\\ &= \frac{5!}{4!\cdot 1!}\\ &= \frac{5\cdot 4!}{4!\cdot 1!}\\ &= \frac{5\cdot 4}{1}\\ &= 5\end{aligned} \qquad \begin{aligned}{}^5C_5 &= \frac{5!}{5!\cdot(5-5)!}\\ &= \frac{5!}{5!\cdot 0!}\\ &= 1\end{aligned}$$

2 **a**

$$\begin{aligned}{}^7C_1 &= \frac{7!}{1!\cdot(7-1)!}\\ &= \frac{7!}{1!\cdot 6!}\\ &= \frac{7\cdot 6!}{1!\cdot 6!}\\ &= \frac{7}{1}\\ &= 7\end{aligned}$$

b

$$\begin{aligned}{}^6C_5 &= \frac{6!}{5!\cdot(6-5)!}\\ &= \frac{6!}{5!\cdot 1!}\\ &= \frac{6\cdot 5!}{5!\cdot 1!}\\ &= \frac{6}{1}\\ &= 6\end{aligned}$$

c

$$\begin{aligned}{}^{12}C_{10} &= \frac{12!}{10!\cdot(12-10)!}\\ &= \frac{12!}{10!\cdot 2!}\\ &= \frac{12\cdot 11\cdot 10!}{10!\cdot 2!}\\ &= \frac{12\cdot 11}{2}\end{aligned}$$

$= 66$

d $$^{8}C_5 = \frac{8!}{5! \cdot (8-5)!} = \frac{8!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot 6}{6} = 56$$

e $$^{100}C_{99} = \frac{100!}{99! \cdot (100-99)!} = \frac{100!}{99! \cdot 1!} = \frac{100 \cdot 99!}{99! \cdot 1!} = \frac{100}{1} = 100$$

f $$^{1000}C_{998} = \frac{1000!}{998! \cdot (1000-998)!} = \frac{1000!}{998! \cdot 2!} = \frac{1000 \cdot 999 \cdot 998!}{998! \cdot 2!} = \frac{1000 \cdot 999}{2} = 499500$$

3 a $$^{n}C_1 = \frac{n!}{1! \cdot (n-1)!} = \frac{n!}{(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = n$$

b $$^{n}C_2 = \frac{n!}{2! \cdot (n-2)!} = \frac{n!}{2 \cdot (n-2)!} = \frac{n \cdot (n-1) \cdot (n-2)!}{2 \cdot (n-2)!} = \frac{n(n-1)}{2}$$

c $$^{n}C_{n-1} = \frac{n!}{(n-1)! \cdot (n-(n-1))!} = \frac{n!}{(n-1)! \cdot 1!} = \frac{n \cdot (n-1)!}{(n-1)!} = n$$

d $$^{n+1}C_1 = \frac{(n+1)!}{1!\cdot(n+1-1)!} = \frac{(n+1)!}{n!} = \frac{(n+1)\cdot n!}{n!} = n+1$$

e $$^{n+2}C_n = \frac{(n+2)!}{n!\cdot(n+2-n)!} = \frac{(n+2)!}{n!\cdot 2!} = \frac{(n+2)\cdot(n+1)\cdot n!}{n!\cdot 2} = \frac{(n+2)(n+1)}{2}$$

f $$^{n+1}C_{n-1} = \frac{(n+1)!}{(n-1)!\cdot((n+1)-(n-1))!} = \frac{(n+1)!}{(n-1)!\cdot 2!} = \frac{(n+1)\cdot n\cdot(n-1)!}{(n-1)!\cdot 2!} = \frac{n(n+1)}{2}$$

4 a There are 10 items and 3 to arrange. This can be done in $10\times 9\times 8 = 720$ ways.

b There are 10 items and 3 to select. This can be done in $^{10}C_3 = 120$ ways.

5 5 objects can be selected from 52 in $^{52}C_5 = 2598960$ ways.

6 a $^{10}C_1 = 10$

b $^{10}C_2 = 45$

c $^{10}C_8 = 45$

d $^{10}C_9 = 10$

7 $^{45}C_7 = 45379620$

8 3 vertices are to be selected from 8. This can be done in $^{8}C_3 = 56$ ways.

9 a This is the same as asking how many ways can 2 teams be selected from 10. This can be done in $^{10}C_2 = 45$ ways.

b Let n be the number of teams. Then,

$$\begin{aligned} ^{n}C_2 &= 120 \\ \frac{n!}{2!\cdot(n-2)!} &= 120 \\ \frac{n\cdot(n-1)\cdot(n-2)!}{2!\cdot(n-2)!} &= 120 \\ \frac{n\cdot(n-1)}{2} &= 120 \\ n(n-1) &= 240 \\ n^2-n-240 &= 0 \\ (n-16)(n+15) &= 0 \end{aligned}$$

$\Rightarrow n = 16 \text{ as } n > 0.$

10 Let n be the number of people at the party. Then

$$\begin{aligned}
{}^{n}C_{2} &= 105 \\
\frac{n!}{2!\cdot(n-2)!} &= 105 \\
\frac{n\cdot(n-1)\cdot(n-2)!}{2!\cdot(n-2)!} &= 105 \\
\frac{n(n-1)}{2} &= 105 \\
n(n-1) &= 210 \\
n^2-n-210 &= 0 \\
(n-15)(n+14) &= 0 \\
\Rightarrow n &= 15 \text{ as } n > 0
\end{aligned}$$

11

$$\begin{aligned}
\text{RHS} &= {}^{n}C_{n-r} \\
&= \frac{n!}{(n-r)!\cdot(n-(n-r))!} \\
&= \frac{n!}{(n-r)!\cdot r!} \\
&= \frac{n!}{r!\cdot(n-r)!} \\
&= {}^{n}C_{r} \\
&= \text{LHS}
\end{aligned}$$

12 Each diagonal is obtained by selecting 2 vertices from n. This can be done in ${}^{n}C_{2}$ ways. However, n of these choices will define the side of the polygon, not a diagonal. Therefore, there are ${}^{n}C_{2} - n$ diagonals.

13 There are ${}^{10}C_{5}$ ways of choosing five students to belong to team A. The remaining five students will belong to team B. However, the labelling of the teams doesn't matter, so we must divide by 2.

14 There are ${}^{12}C_{6}$ ways of choosing six students to belong to team A. The remaining students will belong to team B. However, the labelling of the teams doesn't matter, so we must divide by 2. This gives $\frac{{}^{12}C_{6}}{2} = 462$

15 We begin with the right hand side. This gives,

$$\begin{aligned}
\text{RHS} &= {}^{n-1}C_{r-1} + {}^{n-1}C_{r} \\
&= \frac{(n-1)!}{(r-1)!(n-1-(r-1))!} + \frac{(n-1)!}{r!(n-1-r)!} \\
&= \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!\,(n-r-1)!} \\
&= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left(\frac{1}{n-r} + \frac{1}{r}\right) \\
&= \frac{(n-1)!}{(r-1)!(n-r-1)!}\,\frac{n}{r(n-r)} \\
&= \frac{n!}{r!(n-r)!} \\
&= \text{LHS.}
\end{aligned}$$

16a The number of ways of selecting 3 dots from 25 is ${}^{25}C_{3} = 2300$.

b There are 12 lines of 5 dots. From each of these lines we select 3 from 5 dots. This can be done in $12 \times {}^5C_3 = 120$ ways.

There are 4 lines of 4 dots. From each of these lines we select 3 from 4 dots. This can be done in $4 \times {}^4C_3 = 16$ ways.

There are 16 lines of 3 dots. Note: if you've found only 4 lines, then look harder! From each of these lines we select 3 from 3 dots. This can be done in $16 \times {}^3C_3 = 16$ ways.

Therefore, there are $120 + 16 + 16 = 152$ ways of selecting 3 dots that lie on a straight line.

c The three dots will form a triangle if they don't form a line. So the number of ways of selecting three dots to form a triangle will be the total number of selections, minus those selections that form a line. This gives $2300 - 15223 = 2148$.